3D Computer Vision

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Lecture 18 – Projective Reconstruction
(Chapter 10)

The Structure-from-Motion (SfM) Paradigm

Multiple images of a (rigid) scene.
(Metric) 3D model

Euclidean Structure-from-Motion with Uncalibrated Cameras

1. Pick up one camera (or more…)
2. Take some pictures
3. Compute a Euclidean 3D model of the scene structure and the camera (intrinsic and extrinsic) parameters from those pictures and the following assumptions
   1. The scene is rigid
   2. Some of the camera intrinsic parameters are known or constant

Why Using Uncalibrated Cameras?

- The “classical” approach requires (at least) one image of a calibration object (i.e. of which the Euclidean 3D model is known)
- Camera self-calibration is flexible and efficient
- Does not require a special apparatus
- Does not require the user to interact
- Mild assumptions on the imaging conditions (thus applicable to images and videos coming from unknown sources such as DVDs and the internet)

Some Notations

- \(i\) Image index
- \(n\) Number of images, \(i = 1, \ldots, n\)
- \(j\) Point index
- \(m\) Number of points, \(j = 1, \ldots, m\)

The Coordinate Frame Ambiguity

The basic projection equation is:

\[
q_{ij} \sim K_i (R_i t_i) Q_j
\]

with

\[
Q_j \sim \begin{pmatrix} Q_j^T & \alpha \end{pmatrix}
\]

Let the \((4 \times 4)\) matrix \(D\) represent a Euclidean transformation in 3D space:

\[
D = \begin{pmatrix} R & t \end{pmatrix}
\]

with \(R \in SO(3)\)

Change the projection equation to:

\[
q_{ij} \sim K_i (R_i t_i) DJ^{-1} Q_j
\]

\[
\sim K_i (R_i R - \alpha R t_i)
\]

\[
\sim K_i (R_i t_i) Q_j
\]

This is an equivalent Euclidean 3D model, giving rise to the same projections, but expressed in a different 3D coordinate frame.
The Coordinate Frame Ambiguity

Let us try the same reasoning with \((3 \times 4)\) projection matrices, without preserving the calibration matrices \(K_i\):

\[
q_{ij} \sim P_iQ_j
\]

In this case, any 3D homography represented by a non-singular \((4 \times 4)\) matrix \(H\) can be inserted instead of \(D\):

\[
q_{ij} \sim P_iHH^{-1}Q_j
\]

\((P_i, Q_j)\) and \((P'_i, Q'_j)\) are two equivalent 3D models. They give rise to the same projections but are expressed in different projective coordinate frames. They are projective 3D reconstructions.

Remember: homographies do not preserve angles, length ratios and parallelism.

Different Strata

- Euclidean stratum: \( γ^E \sim D \sim \begin{pmatrix} R & t \\ 0 & 0 & 1 \end{pmatrix} \)
- Metric stratum: \( γ^M \sim \begin{pmatrix} sR & t \\ 0 & 0 & 1 \end{pmatrix} \)
- Affine stratum: \( γ^A \sim \begin{pmatrix} A & t \\ 0 & 0 & 1 \end{pmatrix} \)
- Projective stratum: \( γ^P \sim H \sim \begin{pmatrix} H \bar{h} & h_i \\ 0 & 1 \end{pmatrix} \)

The Link Between a Projective and a Euclidean Reconstruction

Given a projective reconstruction:

\[
q_{ij} \sim P_iQ_j
\]

There exist an upgrading homography, denoted \(Z\), such that:

\[
P_i^E \sim P_iZ \quad \text{and} \quad Q_j^E \sim Z^{-1}Q_j
\]

is a Euclidean reconstruction

Finding \(Z\) using constraints on the cameras is called camera self-calibration.

The Uncalibrated SfM Paradigm
About Conics and Quadrics

★ They are represented by symmetric matrices.
★ The set of points are given by quadratic equations:
  \[ q^T C q = 0 \quad \text{and} \quad Q^T C Q = 0 \]
★ What is the number of degrees of freedom of a conic and of a quadric?
★ How many points to define a conic and a quadric?

Dual Conics and Quadrics

★ \( C \) is a point conic.
★ The dual (line) conic is represented by the adjoint:
  \[ C^* \sim C^{-T} \]
★ The set of tangent lines to \( C \) is defined by:
  \[ l^T C l = 0 \]
★ The dual (plane) quadric is similarly defined.

Transformation of Conics and Quadrics

Under the point transformation \( q' = Hq \), what is the conic transformation rule? What is the dual conic transformation rule?

\[ C' \sim H^{-T} C H^{-1} \]
\[ (C')^* \sim H C^* H^T \]

Conics, Quadrics and Cameras

★ A quadric \( Q \) projects to a conic \( C \) as:
  \[ C^* \sim P Q^* P^T \]
★ A conic \( C \) back project to a viewing cone, represented by a degenerate quadric \( Q_{\text{co}} \):
  \[ Q_{\text{co}} \sim P^T C P \]
★ Note: the (left and right) kernel of \( Q_{\text{co}} \) is the centre of projection and coincides with the vertex of the viewing cone.

Closure

★ 3D reconstruction is possible without knowing the camera intrinsic parameters, but some weak constraints on them.
★ The uncalibrated SfM paradigm is to
  ■ Make a projective reconstruction
  ■ Self-calibrate it
  ■ More flexible than static camera calibration.