L’autocalibrage linéaire d’une caméra à focale variable et ses mouvements critiques

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Outline

1. Flash-back to Self-Calibration of a Camera with “Unknown Varying Focal Length”

2. The Dual Self-calibration Problems

3. Critical Motion Sequences (CMS) in Dual Linear Self-calibration
   - Condition of Camera Motion Criticality
   - Formal Derivation of CMSs
   - Signature Sequences of critical CMSs
   - Identification and Resolution of Artificial CMSs

4. A Simple Test on Real Images

5. Conclusion
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Flash-back to Self-Calibration of a Camera with “Unknown Varying Focal Length”

Reminder of the Facts

Self-calibration paradigm

► “Assumption-free” upgrade of a 3D reconstructed scene from projective to Euclidean.

\[
\Omega_\infty / \pi_\infty
\]

projective reconstruct. + Euclidean structure = Euclidean reconstruct.

► The Euclidean structure is given by the absolute conic at infinity \( \Omega_\infty \) on \( \pi_\infty \).
Self-calibration and critical motions

rotation

translation

+ change of scale

\[ \Omega_\infty \]

\[ \pi_\infty \]
Self-calibration of an “unknown varying focal length” camera

- Camera has *known intrinsics except a time-varying focal length*.


Theoretical Critical Motion Sequences (CMS) in self-calibration

- Those motions which cause *any* of these algorithms to fail.

- Completely known for the “unknown varying focal length” camera [Kahl et al.:00][Sturm:02].
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Theoretical CMS vs. artificial CMS

In addition to theoretical CMS, algorithms may have artificial CMS to deal with.
- Caused by constraints being neglected (e.g., on $\Omega_\infty$ on $\pi_\infty$).

In this talk

We investigate CMS in dual linear self-calibration, as popularized in [Pollefeys et al.:99].

😊 Motivation: easy-to-implement self-calibration algorithm.

The issues at stake are:
- How to derive Euclidean descriptions of artificial CMS?
- How to identify and resolve these artificial CMS?
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The Dual Self-calibration Problems

Projective geometry of 3D quadrics

- **Quadric** → symmetric matrix $Q^* \in S(\mathbb{R}, 4)$.
- **Virtual** quadric if purely imaginary (while $Q^* \in S(\mathbb{R}, 4)$).
- Proper or **degenerate** quadric ($\det Q^* = 0$)
- **Signature** $(p, n)$ with $p \geq n \geq 0$ and $p + n = \text{rank}$

Dual 3D geometry of $\Omega_\infty$ : the Dual Absolute Quadric (DAQ)

In dual 3-space : (degenerate) rank-3 quadric $\Rightarrow$ conic

$\Omega_\infty$ on $\pi_\infty \Rightarrow$ virtual rank-3 quadric

$\rightarrow$ rank-3 symmetric matrix $Q^*_\infty \in S(\mathbb{R}, 4)$

referred to as the DAQ.

$\text{signature}(Q^*_\infty) = (3, 0)$ means “$Q^*_\infty$ is a virtual rank-3 quadric”.
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The Dual Self-calibration Problems

Statements for the ‘Unknown Varying Focal Length’ Camera

Given a camera $P^i$, the absolute conic projects to

\[
Q^*_\infty \overset{P^i}{\longrightarrow} P^i Q^*_\infty P^i \mathbf{T} \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & (f^i)^{-2} \end{pmatrix}.
\]

i.e., to circles, centered at the principal point.

The constrained problem of dual self-calibration

Given a projective motion sequence \( \{P^1, \ldots, P^n\} \), seek a quadric \( X^* \in \mathbb{R}^{4 \times 4} \), such that

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X^* \overset{P^i}{\longrightarrow} P^i X^* P^i \mathbf{T} \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & □ \end{pmatrix}
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subject to constraint,

\[
\text{signature}(X^*) = (3, 0).
\] (1)

The considered unconstrained problem statement: “dual linear self-calibration”

Same problem as above but forget constraint (1).
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Statements for the ‘Unknown Varying Focal Length’ Camera

False DAQ

- A solution to the self-calibration problem \( x^* \neq Q_{\infty}^* \) is called a false DAQ.
- False DAQs only exist in the presence of critical motions [Sturm:02][Kahl et al.:00].

The unconstrained problem ("dual linear self-calibration") introduces:

- additional false DAQS,
- and, hence, additional (so-called artificial) critical motions.
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Fundamental condition of criticality

A motion sequence is critical \textit{iff} there exists a quadric in dual 3-space such that:

1. all camera centres are foci of the quadric;
2. all optical axes are focal axes of the quadric.

If you don’t want that a quadric be a false DAQ:

"DON'T MOVE ON ITS FOCI!"
Definition (Confocal Family)

The **confocal family** of the quadric $x^*$ is the linear family of quadrics

$$x_u^* = x_0^* - uQ_\infty^*, \quad u \in \mathbb{R}.$$

Definition (Foci and Focal conics)

Foci of $x^*$: points of the degenerate quadrics (**focal conics**) of the confocal family of $x^*$.
Fact (Focal axis)

*Focal axis of $x^*$ through a foci = “generating line” of the focal conics of $x^*$.***
Fact (focus and focal axis of a quadric)

Let $X^*$ refer to either a quadric or a conic.

- **Focus of $Q^*$** = vertex of a circular cone tangent to $X^*$.
- **Focal axis through such a focus** = revolution axis of a circular cone tangent to $X^*$. 
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Symbolic computation:

- For each type of Euclidean quadric,
  - Determine its real foci on focal conics and its real focal axes through them.

General
- Central
- Noncentral

Of Revolution
- Subclass 1
- Subclass 2
- Subclass 3
- Spheres
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Critical Motion Sequences (CMS) in Dual Linear Self-calibration

Signature Sequences of critical CMSs

Signature Sequences of critical CMSs

(H) A family \( \{ X^*_v = X^*_1 - vX^*_2 \} \) of false DAQs exists.

Constructing the signature sequence of a CMS

\[ \{ X^*_v = \lambda \mid \text{det} X^*_\lambda = 0 \} \quad \rightarrow \quad \text{Signature sequence algorithm} \quad \rightarrow \quad \{ \cdots, (\cdots (p_\lambda, n_\lambda) \cdots), \cdots \} \]

signature of \( X^*_\lambda \)

signature sequence

Example of signature sequence

\[ \{ (3, 0), (2, 1), (2, 1) \} \times 2, \times 1, \times 1 \]

\[ \Omega_\infty \times 2 \]

\[ \times 1, \times 1 \]
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A CMS can be resolved iff the signature \((3, 0)\) appears only once in the signature sequence.

All artificial CMS can be resolved.
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The “Model House” sequence

- **LS problem:** \[
    \min_{\tilde{X}^*} \left\| \tilde{A}\tilde{X}^* \right\|^2
    \quad \left\| \tilde{X}^* \right\| = 1
\]

- **Nonunique solution:** \[ \text{dim}(\text{null} A) = 2 \]

- Computed signature sequence:
  \[
  \{ (3, 0), (((1, 0))) \}
  \]

  This means “artificial criticality wrt concentric spheres”

- Solution disambiguated = succeeded!
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A well-founded add-on to the DAQ formalism for self-calibration

Three take-home messages

- **Theory**: Condition of criticality describing both theoretical and artificial CMSs.
  - 1st message: "If you don’t want that a quadric be a false DAQ ....
    - ... don’t move on its foci!"

- **Algorithm**: Classification of CMSs via a projective descriptor.
  - 2nd message: All CMSs can be uniquely identified.

- **Algorithm**: Resolution by a posteriori enforcing the DAQ signature.
  - 3rd message: All artificial CMSs can be resolved.